**Horseshoe and Spike-and-Slab Priors**

* **Connection between horseshoe and spike-and-slab prior**
  + The horseshoe prior consists of a normally distributed local shrinkage parameter nested as the mean of a normally distributed variable with global variance. This gives heavy tails and a pdf that approaches infinity as the variable approaches 0
  + The spike-and-slab prior consists of two separate priors – one a point mass (or very narrow normal distribution) about 0 for coefficients that are 0 (i.e. unselected), and a wide normal distribution about 0 for coefficients that are not 0 (i.e. selected)
  + If we consider the formulation of spike-and-slab that uses a very narrow normal distribution instead of a point mass, perhaps we could view horseshoe as some sort of limiting case, at least near 0?
  + Seems to be an intuitive relationship between the two, but would need to analyze more rigorously
    - Find more explicit mathematical connection
* **Computational efficiency of horseshoe prior**
  + Paper claims that for ~10^6 parameters, ~10^3 observations, horseshoe-probit regression takes ~2 minutes (w/ MCMC sampling), which would likely scale up for more parameters (on the order of 10^7 SNPs, perhaps) without taking extremely long
  + Can be even faster if one only obtains point estimates, but it is probably desirable to retain the probabilities for variable selection decisions, other uses
    - Why do they still use MCMC sampling?
    - Is anyone using greedy algorithm/looking for local optima with horseshoe prior?
* **Decision rule**
  + , plays the role of a shrinkage weight and a “pseudo posterior inclusion probability”
  + Proposed decision rule: reject null of if
  + If we can find a rigorous connection between the horseshoe and spike-and-slab priors, then this pseudo probability should be very close to the posterior inclusion probability
    - Given approximate PIP, can do Bayesian false discovery rate control (similar: local false discovery rate control, Efron)